Short Paper

Selection of schools in districts using a bi-objective MILP model: A case study from Paraguay

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ABSTRACT

The district of Abaí in the department of Caazapá, Paraguay, has building infrastructure conditions that can be improved in order to improve the quality of education. The reduction of schools could contribute to better management. Unfortunately, any change in the operation of the system generates strong resistance to change when it comes to closing schools. We present a bi-objective model that simultaneously seeks to reduce the total cost of the education system and minimize the number of students displaced from schools that must be closed.

KEYWORDS

Schools, Facility opening and closing, Infrastructure resizing, Mixed linear programming, Bi-objective programming.

1 INTRODUCTION AND BACKGROUND

The quality of the educational infrastructure can influence the development of the educational process [15]. Several authors, [8] cite studies that demonstrate a direct relationship between the performance of students and the configuration of the physical space in which they are, determined by the quality of the design of the spaces and their adequate conservation [12][6][9].

The educational infrastructure in Caazapá, Paraguay, is generally poor, and the district of Abaí is no exception. In 2021, 83% of the district's establishments requested investment in infrastructure (repairs or new construction). Not all schools have basic services, 47% do not have drinking water service, and 5% do not have electricity. Furthermore, in 2021 77% of schools requested furniture (tables and desks), while 84% requested the repair of their classrooms.

In Paraguay, public spending on education is low and, most of the budget, covers fixed costs. In 2019, 92 % of the budget was allocated to salaries.

Public education buildings require a maintenance plan to function properly throughout their useful life of 30 years. In addition to every dollar spent in the school construction, 5 dollars are spent for its operation ([14][1] cited by [8]).

From an economic point of view, the consolidation of demand could be a robust solution to reduce the operating and maintenance costs of schools, and at the same time, to reduce the number of establishments, allowing a better focus of resources for investment and improvement of schools.

Even knowing this, it is not at all possible to design the system from scratch. Modifying any aspect of the educational system requires the consensus of several sectors with different interests[5], and a strong change resistance can be assumed as a common denominator. For example, in 2019, the teachers' union rejected the measure taken by the Ministry to eliminate sections due to the low number of enrolled students, arguing that the right to education access was violated.[4]

Some studies explain the consequences of school closings, but none mention that a negative impact on student grades is directly attributable to this fact. However, they describe some undesirable effects on the community.

In the short term, school closures can improve the performance of students displaced from low-performing schools to better-performing schools. In the long term, it can cause variation in student performance in host schools; other effects are the redistribution of students and teachers in the district and the resistance to change [6].

This work seeks to offer solutions that minimize the total cost of the educational system and reduce the proposal's impact on the affected students, seeking both the well-being of the students (social objective), and the economic feasibility of the application of the proposal (economic objective).

A bi-objective mixed-integer linear programming model is used to minimize the total cost and at the same time minimize the number of students assigned to a different school due to the schools closure. We used the parametrization technique to solve the model that was optimized with Cplex to find the Pareto optimal set values. This is the first model to propose this type of approach to select schools in Paraguay to the best of our knowledge.

2 LITERATURE REVIEW

In the literature, many studies seek to minimize costs and improve system performance by considering different performance measures: decrease response times, increase coverage, reduce environmental impact, etc. Here are some works that address these types of problems using the bi-objective optimization approach.

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In 2019 [16], developed a bi-objective spatial optimization model to minimize the total number of stations located and the distance / weighted travel time to places that request services, prioritizing the location in high-risk areas. They presented a constraint-based solution procedure to generate the Pareto frontier. The model was applied to optimally locate fire stations in the city of Nanjing, China.

In 2017 [10], formulated an integer programming model to determine the number of trucks needed to collect hazardous materials and the routes these trucks must follow, seeking to minimize the total population exposed to dangerous materials and the transport cost. This work presents a novel approach as it allows different hazardous materials to be loaded on the same truck. They used the weighting method to find the set of non-dominated solutions, where both objectives are normalized.

In 2015 [9], plan to offer balanced solutions between reducing waiting times for patients who are candidates for elective surgery and the costs of added planning associated with the care of these patients. The authors develop a bi-objective model to optimize the conflicting objectives simultaneously and apply the non-dominated genetic classification algorithm. As a result, they offer a solution set that satisfies the proposed objectives in different proportions.

In 2012, [13], using a bi-objective non-linear programming model, sought on the one hand, to minimize the costs of implementing the modernization of a water supply system for a brewery, and on the other hand, to minimize the environmental impact by comparing environmental indicators and reference values. They offer options on achievable benchmarks for each course of action in water network design.

In 2010, [11], studied the territory design of a bottled beverage distribution company. They approached it using a bi-objective optimization model that sought to divide a set of geographic units into a fixed number of territories to minimize the dispersion and the balance concerning to the number of clients. They proposed an improved ϵ-constraint method to generate the Pareto optimal frontier.

3 SCHOOL ESTABLISHMENT SELECTION PROBLEM.

The problem of selecting schools in this study is to select from a set of schools, which ones should remain open, which should be closed, and how to reassign their students. In addition, the cost of installing each type of school in each place depends on the investments necessary to adapt that school to the infrastructure requirements for its qualification.

3.1 Methodology

For the design of the mathematical model, constraints were adapted and added to the model presented in [2], using the first objective function that represents the total cost of the educational network.

The total cost of the education system includes the operating costs, maintenance, and investment in the infrastructure of schools and transportation costs for students.

We define a student's home establishment as the establishment they currently attend.

The objective function is to minimize the total cost of operating the education system and to minimize the number of students displaced from their establishment of origin.

We use the parametrization technique to solve the bi-objective model. We assign weights to the objective functions to solve a model with a single objective solution, composed of the weighted sum of the original functions.

$$Z = \alpha\, f1 \;+\; (1-\alpha)\, f2$$

Where:

$$f1 = \sum_{l}\sum_{t} cost_{lt} X_{lt} + \sum_{a}\sum_{l} costbus Z_{al} + \sum_{a}\sum_{l} csend2_{al} Z_{al}$$
$$+ \sum_{a}\sum_{l \in GO_a}\sum_{t \in in_l}\sum_{g \in ed_t} csend_{al}\, pop_{ag}\, Y_{aglt}$$
$$+ \sum_{l}\sum_{t \in in_l}\sum_{g \in ed_t} costadd_{lgt} ADD_{lgt}$$
$$+ \sum_{l}\sum_{t \in in_l}\sum_{g \in ed_t} costsub_{lgt} SUB_{lgt}$$
$$+ \sum_{l} costconst_l CONS_l$$
$$+ \sum_{l}\sum_{t \in in_l}\sum_{g \in ed_t} penalty\ _{lgt} S_{lgt}$$

$$f2 = \sum_{l}\sum_{t \in Closed} mat_l X_{lt}$$

To make the values of the objective functions comparable, we perform the normalization of both functions, following the methodology used by [3] and [10].

$$z_i = \left[\frac{f_i - I_i}{AI_i - I_i}\right]$$

where Z_i is the normalized objective (cost or displaced students), f_i is the objective function before normalization, I_i is the best value of f_i, and AI_i is its worst value.

This model, in addition to considering a second social objective function that seeks to minimize the displacement of students, has the following novelties:

There are two options for transporting students for each school. In the first, transportation to the school is at the expense of the student, who may receive a grant for this travel. In the second alternative the district arranges for transportation from bus stops to schools. One of these options is defined as a parameter for each establishment.

Schools can operate in two shifts. The number of shifts is defined in the type of establishment. Therefore, it is necessary to separate the costs of adding sections, which only includes the cost of teaching, from the costs of building a classroom. In this regard, the model is like that present[7].

There are no predefined sizes. For each school, we start from an initial configuration from the current operation of each establishment, and sections can be added or eliminated at any level according to the classrooms capacity.

3.2 Model specification

In Table 1 we present the index and subsets of index used in the model. Next, Table 2 shows the definition of the model variables. Finally, Table 3 shows the parameters.

According to the definitions made previously, the objective function is expressed as follows:

$$Minimize \quad Z = \alpha z_1 + (1-\alpha)\, z_2$$

Subject to the following restrictions:

Table 1: Index and subsets of the model.

Index	Description
a	Student stops (current school)
c	Districts of the department of Caazapá
l	School establishment to which a student may be assigned
g	School grades
g'	Multigrade school grades
t	Teaching modality of the establishments
$t \in in_l$	School modalities that are options in the establishment "l"
$g' \in grade_{gt}$	Degrees that can be combined with a grade "g" for schools of modality "t"' (in case of having multigrade)
$a \in SA_c$	Set of stops "a" that are part of district "c"
$l \in SL_c$	Set of establishments "l" that can be chosen in district "c"
$l \in GO_a$	Set of establishments "l" that are options for students of origin "a"
$a \in GO_l$	Set of schools "a" that can send students to school "l"
$g \in ed_t$	Set of school grades "g" that can be taught in a "t" mode school

(1) All students must be assigned to a school.

$$\sum_{l \in GO_a} \sum_{t \in in_l} Y_{aglt} = 1;\ \forall a,\ \forall g \in ed_t$$

(2) A school only works in one of the modalities available for that establishment.

$$\sum_{t\ \in\ in_l} Y_{lt} = 1;\ \forall\ l$$

(3) Limit to the assignment of students according to the available capacity - Multigrade case.

$$\sum_{a \in GO_l} \sum_{g' \in grade_{lt}} \frac{pop_{ag'} Y_{aglt}}{csize_{lg't}} \leq \sum_{g' \in grade_{gt}} class_{lg't} X_{lt} + \sum_{g' \in grade_{gt}} ADD_{lg't} - \sum_{g' \in grade_{gt}} SUB_{lg't} + Slgt$$
$$\forall t = 3, \forall\ l, g'$$

(4) Limit to the assignment of students according to the available capacity - Monograde case.

$$\sum_{a \in GO_l} \frac{pop_{ag} Y_{aglt}}{csize_{lgt}} \leq class_{lgt} X_{lt} + ADD_{lgt} - SUB_{lgt} + S_{lgt}$$
$$\forall l, \forall g, \forall\ t \in 1, 2$$

Table 2: Decision variables

Variable	Description
$X_l t$	Binary variable that equals one if a school of modality "t" works in the establishment "l", and 0 otherwise
$Y_a glt$	Fraction of students from school "a", grade "g" attending school "l" operating in mode "t"
$Z_a l$	Binary variables that equals one if there is an assignment of students from establishment "a" to destination «l» and 0 otherwise
$ADD_l gt$	Number of sections to be opened
$SUP_l gt$	Number of sections to be eliminated
$CONSl$	Number of classrooms to be built in the establishment "l"
$S_l gt$	Extra percentage of students to add to the existing courses of grade "g" in the school of modality "t" in-destination "l"

(5) Limit of sections to add according to the number of available classrooms.

$$\sum_g ADD_{lgt} \leq n_{lt} X_{lt} + cturn_t CONS_l + \sum_g SUB_{lgt}; \forall l,\ t \in in_l$$

(6) You cannot eliminate courses in unselected establishments.

$$SUB_{lgt} \leq X_{lt} class_{lgt};\ \forall l, t, g$$

(7) Limit of classrooms to be built according to the physical space available.

$$CONS_l \leq X_{lt} espdisp_l; \forall l, t$$

(8) Limit of students for establishments that operate in the multigrade modality.

$$\sum_a \sum_g pop_{ag} X_{aglt} \leq multmax;\ \forall l,\ t = 3$$

(9) Limit for slack. This slack can only exist in an establishment course if there are sections of that grade.

$$X_{lt} class_{lgt} + ADD_{lgt} - SUB_{lgt} \leq \frac{S_{lgt}}{lim_{lgt}};\ \forall l, g, t$$

(10) Students cannot be assigned to establishments that operate in incompatible modalities.

$$\sum_{t \notin in_l} Y_{aglt} = 0,\ \forall\ a, g, l$$

(11) Students cannot be assigned to grades incompatible with the modality of a selected establishment.

$$\sum_{g \notin ed_t} Y_{aglt} = 0,\ \forall\ a, l, t$$

(12) Maximum travel distance for students.

$$dist_{al} Z_{al} \leq distmax_a,\ \forall\ a, l$$

Table 3: Decision variables

Parameters	Description
$cost_{lt}$	Operational cost of the establishment.
$costadd_{lgt}$	Cost of adding a section in the establishment.
$costsub_{lgt}$	Cost of removing a section in an establishment.
$costconst_{l}$	Cost of building and equipping a classroom.
$csend_{al}$	Transportation cost of each student from the bus stop to the establishment.
$csend2_{al}$	Transportation cost incurred if at least one student is assigned from a stop to a establishment.
$penalty_{lgt}$	Penalty for exceeding the nominal capacity of a class.
$class_{lgt}$	Capacity of the establishments for each type of teaching.
n_{lt}	Total number of sections that can be added to a school for each teaching modality.
$limit_{lgt}$	Maximum percentage to increase in the capacity of courses for establishments of each teaching modality.
$multmax$	Maximum number of students that a multigrade school can have.
$csize_{lgt}$	Ability to serve students in a section.
$espdisp_{l}$	Available space to build in the establishment.
$cturn_{t}$	Number of shifts that can exist in a school.
pop_{ag}	Number of students of each grade in the establishments.
$cierre_{max}$	Maximum number of establishments that can be closed.
$maxdist_{al}$	Maximum distance a student can travel.
$dist_{al}$	Distance between each establishment and their respective stops.
$costbus$	Investment necessary to buy a means of transport.
$cpbus$	Maximum capacity of the means of transport.

(13) Bus capacity limit.

$$cpbus_{al}\ Z_{al} \geq \sum_{t} \sum_{g} Y_{aglt}\ pop_{ag},\ \forall\ a,\ l$$

(14) Nature of the variables.

$$X_{aglt} \geq 0\ ;\quad \forall\ a, g, l, t$$

$$CONS_{l} \geq 0\ y\ entero\ ;\quad \forall l$$

$$ADD_{lgt} \geq 0,\ SUB_{lgt} \geq 0,$$

$$S_{lgt} \geq 0\ and\ intengers\ ;\quad \forall\ l, g, t$$

$$Y_{lt} \in \{0, 1\}\ ;\quad \forall\ l, t$$

$$Z_{al} \in \{0, 1\}\ ;\quad \forall\ a, l$$

4 CASE STUDY

We solve the model for the district of Abaí in the Department of Caazapá. This district has 85 establishments distributed in 1,547 km2, and for the 2018 school year, they enrolled 907 students.

We define a degree as a course equivalent to one school year, part of the Paraguayan formal education levels. The grades included are pre-kindergarten, kindergarten, and preschool corresponding to the initial level, the nine grades from first to ninth of primary school education, and the three grades from first to the third year of secondary education.

The type of school defines whether the establishment is open or not, the number of shifts, and the kind of teaching (monograde or multigrade). The possible combinations are levels separated in two shifts (monograde), two or more levels together in two shifts (multigrade), levels separated in a single shift, and closed.

In this document, only schools of public administration are considered eligible options. Private or subsidized management schools are not eligible, but have a demand that must be met. Finally, schools that operate in indigenous communities are excluded because their students cannot be reassigned.

For the resolution of the model, the district was divided into two zones. A maximum travel distance of 3 km per student was also established. The distribution of establishments with their respective associations can be seen in Figure 1.

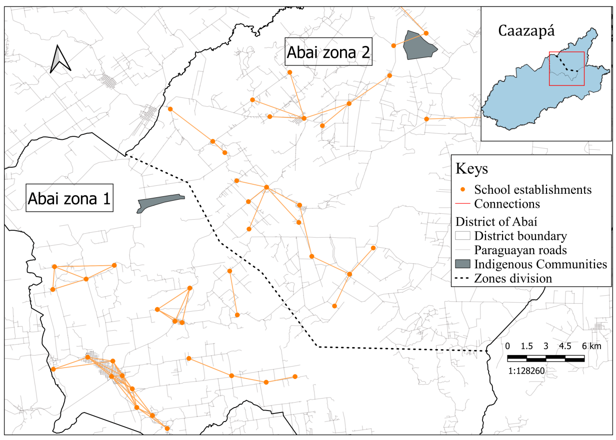

Figure 1: School establishments in the Abaí district. Image prepared by the authors with data from MEC and DGEEC Commons.

Table 4: Non-inferior solutions Abaí zone 1

α	Displaced students	Cost ($)	Execution time (sec)
1	1217	2033313	1016.08
0.96	1217	2033313	428.48
0.92	901	2135780	188.33
0.88	625	2277534	69
0.84	276	2525121	10.09
0.8	163	2632361	6.83
0.76	148	2652496	8.02
0.72	40	2827086	13.19
0.68	40	2827086	11.75
0.64	8	2900772	12.89
0.6	0	2924060	7.66

5 OUTCOMES

We program and solve the model using the IBM ILOG Cplex Optimizer V 12.6 mathematical programming environment with an HP 245 G5 notebook with an AMD A8-7410 APU processor (2.20 GHz) and 8.00 GB RAM.

We solve the model with different α values. In Table 4, we can see that the largest number of displaced students is obtained for zone 1 (1217) when the total cost is optimized. As the value of α decreases, the cost increases, and the number of displaced students is reduced. For α values lower than 0.6, the number of displaced students is 0, and the highest total cost that can be incurred is obtained.

On the other hand, in Table 5. we can see that for zone 2, when the total cost is optimized, the highest number of displaced students is obtained (1201). As in zone 1, as the value of α decreases, the cost increases and the number of students displaced decreases, showing that the objective functions are in conflict.

Figure 2 shows the result with the optimized total cost, considering α equal 1 as scenario 1 for Abaí.

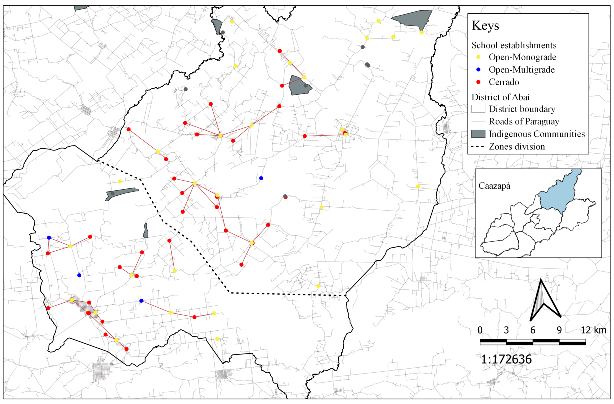

Figure 2: Selected establishments in the district of Abaí-Scenario 1. Image prepared by the authors with data from MEC and DGEEC.

Figure 3 shows the result minimizing the number of displaced students for α equal to 0 as scenario 2 for Abaí.

In Figure 4. and Figure 5. we can see the set of Pareto optimal solutions for both district areas. The points represent a combination between the different target weights and a different distribution of students among the selected schools. In both cases, there is a conflict of objectives where the improvement of one implies the detriment of the other.

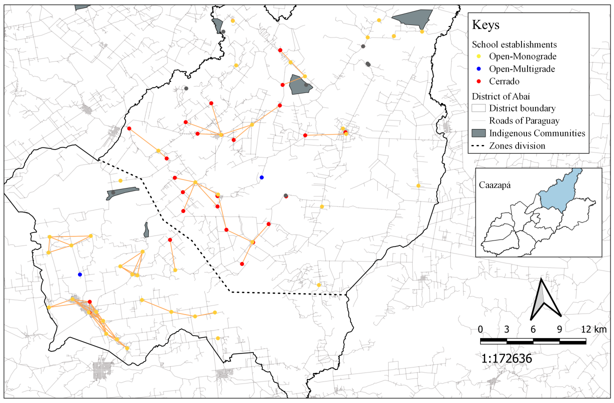

Figure 3: Selected establishments in the district of Abaí-Scenario 2. Image prepared by the authors with data from MEC and DGEEC.

Table 5: Non-inferior solutions Abaí zone 2

α	Displaced students	Cost ($)	Execution time (sec)
1	1201	972870	385.02
0.96	867	987283	428.48
0.92	659	1025300	149.66
0.88	444	1104218	101.61
0.84	308	1179951	7.23
0.8	308	1179951	3.38
0.76	118	1349710	2.41
0.72	89	1384991	2.97
0.68	54	1438782	2.56
0.64	18	1503611	2.92
0.6	18	1503611	3.5
0.56	10	1524266	3.78
0.52	0	1554200	3.94

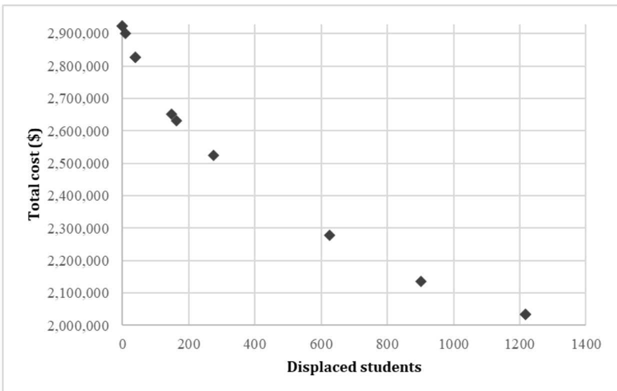

Figure 4: Trade-off curve for Abaí zone 1

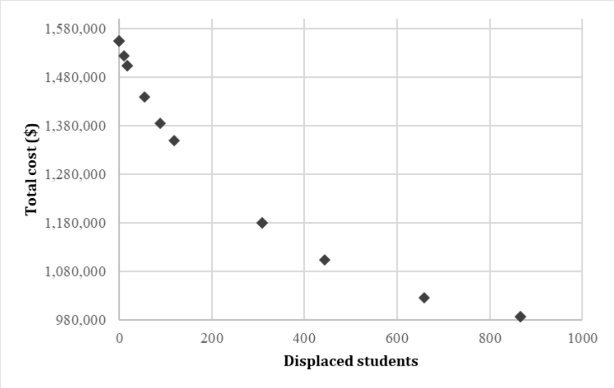

Figure 5: Trade-off curve for Abaí zone 2.

6 CONCLUSIONS

In this article, we present a bi-objective approach to school selection. First, with an economic objective we seek to minimize the total cost of the educational system (operating costs, investment costs and maintenance of the building infrastructure and transportation costs), then we include a social objective seeking to minimize the number of students displaced from closed schools in the proposed school selection. The model was applied to the district of Abaí in the department of Caazapá, Paraguay, for the selection of schools. As a result, a set of solutions is obtained in which the social impact of the proposal is reduced, measured in the number of displaced students, incurring different levels of costs.

In this study, we offer acceptable and flexible solutions for decision makers. The criteria used to measure the social cost of the proposal could be improved or replaced, as well as other qualitative criteria could be included, such as maximizing the number of students assigned to high-performing schools.

For future jobs, vehicle routing can also be used to determine bus routes from stops to schools.

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