Efficient regular path query evaluation using path indexes

George H. L. Fletcher
Eindhoven University of
Technology
The Netherlands
g.h.l.fletcher@tue.nl

Jeroen Peters
Eindhoven University of
Technology
The Netherlands
j.peters.1@student.tue.nl

Alexandra Poulovassilis
London Knowledge Lab
Birkbeck, University of
London, UK
ap@dcs.bbk.ac.uk

ABSTRACT

We demonstrate the use of localized path indexes in generating efficient execution plans for regular path queries. This study is motivated by both the practicality of this class of queries and by the current dearth of scalable solutions for their evaluation. Our proposed solution leverages widely available relational database technology and is often orders of magnitude faster than currently known approaches. We aim in this hands-on demonstration to both highlight the promise of our approach and to stimulate further discussion and study of engineering solutions for this practical yet challenging class of graph queries.

1. INTRODUCTION

Massive graph-structured data collections are ubiquitous in contemporary data management scenarios such as social networks, linked open data, and chemical compound databases. A fundamental paradigm in graph query languages are the so-called *regular path queries (RPQs)* [16]. RPQs specify a regular expression over the edge labels in a graph, and the query answer consists of every path in the graph such that the sequence of edge labels along the path forms a word in the language recognized by the regular expression. Variations and extensions of RPQs are supported in recent query languages such as SPARQL 1.1 [8] and the Cypher language of the Neo4j graph database.[1]

State of the art. Indexing of paths occurring in data has been shown to be effective for query processing in the context of object-oriented and semistructured databases [1, 15]. To our knowledge, however, there has been no investigation of using path indexing for the evaluation of RPQs over graph databases. In particular, three general approaches to RPQ evaluation have been proposed in the literature:

1. Automata- and search-based processing (e.g., [5, 10, 13]), where queries are evaluated by strategies such

---

[1] http://neo4j.com

as breadth-first-search pattern matching of the query graph on the data graph;

2. Datalog-based processing (e.g., [3, 17]), where the Kleene star operator is translated into recursive Datalog programs or recursive SQL views;

3. Reachability-index-based processing (e.g., [6]), where restricted uses of Kleene star are translated into reachability queries, which are then evaluated using off-the-shelf reachability indexes.

Contributions. We present an overview of our ongoing study of the use of path indexes for generating efficient execution plans for RPQs. Our approach supports the evaluation of arbitrary RPQs (unlike approach (3) above), and exhibits significant improvement (often by several orders of magnitude) in query processing times over approaches (1) and (2).

In the next section we briefly define the problem. In Section 3 we introduce the main data structures used in our solution. We then discuss query plan generation and execution in Section 4. We conclude in Section 6 with an overview of our system demonstration.

2. PRELIMINARIES

2.1 Data Model

We consider finite, directed, edge-labeled graphs. A *graph vocabulary* is a finite non-empty set $\mathcal{L}$ of *edge labels* drawn from some universe of labels. Let N be an infinite universe of atomic data objects. An *edge relation* is a finite subset of $N \times N$. A *graph* over vocabulary $\mathcal{L}$ is an assignment G of an edge relation ℓ^G to each $\ell \in \mathcal{L}$, i.e., there is an edge labeled ℓ from m to n if $(m, n) \in \ell^G$. In the sequel, we will sometimes denote this by $\ell(m, n)$ or $m \xrightarrow{\ell} n$. As an example, a graph G_{ex} over the vocabulary $\{\mathsf{supervisor}, \mathsf{knows}, \mathsf{worksFor}\}$ is shown in Figure 1.

The *node set* of G is the collection $nodes(G) = \{n \mid \exists m \in N, \ell \in \mathcal{L} : (n, m) \in \ell^G \text{ or } (m, n) \in \ell^G\}$. For example, the node set of the example graph contains nine elements: $nodes(G_{ex}) = \{\mathsf{ada}, \mathsf{jan}, \dots, \mathsf{zoe}\}$.

Let k be a natural number. If $k = 0$, we say there is a *k-path from s to s*, for every $s \in nodes(G)$. If $k > 0$, for $s, t \in nodes(G)$, we say there is a *k-path from s to t* if there exist $n_0, \dots, n_k \in nodes(G)$ and edge labels $\ell_1, \dots, \ell_k \in \mathcal{L}$ such that $n_0 = s$, $n_k = t$, and, for $0 < i \leq k$, $(n_{i-1}, n_i) \in \ell_i^G$ or $(n_i, n_{i-1}) \in \ell_i^G$.
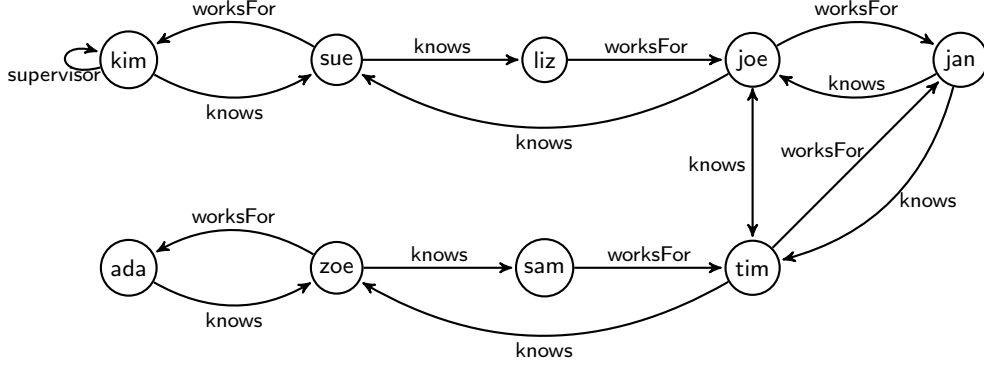
Figure 1: A graph G_{ex} over vocabulary $\mathcal{L} = \{\text{supervisor}, \text{knows}, \text{worksFor}\}$.

We denote by $paths_k(G)$ the set of all pairs of nodes $(s, t) \in nodes(G) \times nodes(G)$ such that there is an i-path from s to t, for some $i \leq k$.

As an example, in the graph G_{ex} we have that $(\text{sam}, \text{ada}) \in paths_2(G_{ex})$ via the paths $\text{sam} \xleftarrow{\text{knows}} \text{zoe} \xrightarrow{\text{worksFor}} \text{ada}$ and $\text{sam} \xleftarrow{\text{knows}} \text{zoe} \xleftarrow{\text{knows}} \text{ada}$, but $(\text{sam}, \text{ada}) \notin paths_1(G_{ex})$.

2.2 Regular Path Queries

Fix a vocabulary $\mathcal{L}$. A *regular path query* (RPQ) is a regular expression R over the alphabet $\{\ell, \ell^- \mid \ell \in \mathcal{L}\}$, i.e., R is generated by the grammar

$$R \quad ::= \quad \epsilon \mid \ell \mid \ell^- \mid R \circ R \mid R \cup R \mid R^{i,j}$$

for $\ell \in \mathcal{L}$ and natural numbers i and j where $i \leq j$. Intuitively, "ϵ" is the identity transition, "ℓ" is forward navigation along an edge with that label, "ℓ^-" is backwards navigation, "$\circ$" is path composition, "$\cup$" is path disjunction (i.e., union of paths), and "$R^{i,j}$" is bounded path recursion.

Given graph G over $\mathcal{L}$, the semantics of evaluating an RPQ R on G is a set $R(G)$ consisting of all pairs (a, b) in $paths(G)$ such that there exists a path from node a to node b in G whose label sequence $\ell_1 \cdots \ell_n$ defines a word in the regular language specified by R.

As examples, we have in the graph G_{ex} of Figure 1 that:

$$\text{supervisor} \circ \text{worksFor}^-(G_{ex}) \quad = \quad \{(\text{kim}, \text{sue})\}$$

and

$$(\text{supervisor} \cup \text{worksFor} \cup \text{worksFor}^-)^{4,5}(G_{ex}) =$$
$$\{(\text{kim}, \text{kim}), (\text{kim}, \text{sue}), (\text{sue}, \text{kim}), (\text{sue}, \text{sue}),$$
$$(\text{ada}, \text{zoe}), (\text{ada}, \text{ada}), (\text{zoe}, \text{ada})\}.$$

Note that we deviate from the traditional syntax of regular expressions by replacing Kleene star "$*$" with bounded recursion. This is motivated by the following two observations. First, bounded recursion is supported (and encouraged) in practical graph query languages such as Neo4j's Cypher.[2] Second, since we focus in this work on index construction and use, we are interested in query evaluation on a given graph. It is easy to establish that for any graph G there exists a natural number $n(G)$ such that for every RPQ R it is the case that $R^*(G) = R^{0,n(G)}(G)$.

3. INDEXES AND HISTOGRAMS

Given a graph G and a fixed $k > 0$, we now present our indexing and selectivity estimation approaches for paths in G localized to neighborhoods of size k.

3.1 k-path indexing

A *label path* is a sequence $\mathsf{p} = \ell_1 \cdots \ell_n$, where $n > 0$ is the *length* of p, and $\ell_i \in \{\ell, \ell^- \mid \ell \in \mathcal{L}\}$ for each $1 \leq i \leq n$.

Our index on G is based on an ordered dictionary (which can be implemented, for example, as a B+tree). In particular, we index $paths_k(G)$ using an ordered k-*path index* $I_{G,k}$ having search key $\langle label\ path, sourceID, targetID \rangle$. Specifically, for each label path p of length at most k, and for each pair of nodes $(a, b) \in \mathsf{p}(G)$, we insert (p, a, b) into $I_{G,k}$. Given a non-empty prefix p of a search key, $I_{G,k}$ returns an ordered list $I_{G,k}(p)$ of all matching entries.

EXAMPLE 3.1. *In the graph of Figure 1, we have*

$$I_{G,k}(\langle \text{knows} \cdot \text{knows} \cdot \text{worksFor} \rangle) =$$
$$\langle (\text{ada}, \text{tim}), (\text{jan}, \text{ada}), (\text{jan}, \text{jan}), (\text{jan}, \text{kim}),$$
$$(\text{joe}, \text{ada}), (\text{joe}, \text{jan}), (\text{joe}, \text{joe}), (\text{kim}, \text{joe}),$$
$$(\text{tim}, \text{jan}), (\text{tim}, \text{kim}), (\text{tim}, \text{tim}) \rangle,$$

$$I_{G,k}(\langle \text{knows} \cdot \text{knows} \cdot \text{worksFor}, \text{jan} \rangle) =$$
$$\langle (\text{ada}), (\text{jan}), (\text{kim}) \rangle,$$

$$I_{G,k}(\langle \text{knows} \cdot \text{knows} \cdot \text{worksFor}, \text{jan}, \text{ada} \rangle) \quad = \quad \langle () \rangle,$$

$$I_{G,k}(\langle \text{knows} \cdot \text{knows} \cdot \text{worksFor}, \text{jan}, \text{joe} \rangle) \quad = \quad \langle \rangle.$$

We have developed a prototype k-path index implementation that leverages the B+ tree index support of PostgreSQL[3]. We translate RPQs into equivalent SQL statements over $I_{G,k}$ implemented as a relational table and backed by a B+tree (see [12] for full implementation details). In building on mature relational technologies, we are following an emerging trend in this direction [4, 7, 9] with practical benefits such as simplicity, ease of integration with and deployment within existing IT ecosystems, and leveraging field-proven technologies.

Notwithstanding the fact that we have described here an implementation of our proposed k-path indexing technique

using existing RDBMS technologies, other recent work [14] describes an implementation of a B+tree-based k-path index "from scratch", focusing on issues such as index size, compression and performance, and undertaking a comparative performance study with the Neo4j graph DBMS over several real and synthetic datasets and query workloads. That evaluation too demonstrates the potential of our k-path indexing approach (showing speed-ups in query evaluation times ranging from 2 times to 8,000 times faster compared with Neo4j). Detailed performance comparison between these approaches to path indexing is an area of future work.

3.2 k-path histogram

For query plan generation over $I_{G,k}$, it is useful to have a data structure $sel_{G,k}$ which, given a label path p of length at most k, returns an estimate of the *selectivity* of p in G, i.e., the fraction of paths in $paths_k(G)$ which satisfy p. As an example, we have in the graph G_{ex} of Figure 1 that $sel_{G_{ex},2}(\mathsf{supervisor} \circ \mathsf{knows}) \approx 0.02$, since only one of the 53 paths in $paths_2(G_{ex})$ is in $\mathsf{supervisor} \circ \mathsf{knows}(G_{ex})$.

There is a rich literature on statistics for query optimization [2]. Here, we adopt the well-established histogram data structure, since it is easy to deploy and extremely successful in practice. In particular, we implement $sel_{G,k}$ as an *equi-depth histogram*. The basic idea here is, given space to store cardinality information about B label path ranges, we keep track of the cardinality of label paths in the graph falling into B contiguous ranges, as induced by their lexicographic order (i.e., in the order maintained in $I_{G,k}$); the ranges are selected such that they each have roughly the same cardinality. We then estimate the cardinality of any given label path by dividing the cardinality of the range in which it occurs by the number of label paths in that range. As a simple example, suppose $k = 1$, $B = 2$, and we have edge labels a, b, and c, with cardinalities 2, 4, and 6, resp. Then the first range covers a and b and the second range covers c, with cardinalities 6 and 6, resp. Using this histogram, the estimated cardinality of a is $\frac{6}{2} = 3$.

As with the path indexes, in our prototype implementation we store and access our histogram as a PostgreSQL table; see [12] for full implementation details.

4. QUERY EVALUATION WITH PATH INDEXES

The processing of a RPQ R proceeds in three steps: The **first step** is to replace each occurrence of bounded recursion in R as a union over its expansion. The result is a semantically equivalent query R' involving only edge labels or their inverses, compositions, and unions. In the **second step**, all unions in R' are "pulled up" to the top level of the query, resulting in a semantically equivalent query R'' consisting of a union of expressions each free of unions and bounded recursion, i.e., $R'' = R_1 \cup \cdots \cup R_n$ where each R_i is a label path. In the **third step**, each disjunct R_i is processed in turn, with the aim of generating a physical execution plan for each R_i in which a merge-join is used whenever possible (to make the best use of the physical sort order of the index) and a hash-join is used otherwise.

As an illustrative example, consider the query $R = \mathsf{k} \circ (\mathsf{k} \circ \mathsf{w})^{2,4} \circ \mathsf{w}$, where k and w abbreviate knows and worksFor, resp. Query plan generation proceeds as follows, where for clarity of presentation we drop explicit use of the concate-

nation operation $\circ$:

1. $(\mathsf{kw})^{2,4}$ is expanded, giving

$$R' \quad = \quad \mathsf{k}(\mathsf{kwkw} \ \cup \ \mathsf{kwkwkw} \ \cup \ \mathsf{kwkwkwkw})\mathsf{w}.$$

2. Nested unions are pulled up to the top level, giving

$$R'' \quad = \quad \mathsf{kkwkww} \ \cup \ \mathsf{kkwkwkww} \ \cup \ \mathsf{kkwkwkwkww}.$$

3. Finally, physical execution plans are generated for each of the disjuncts of R''. In particular, suppose that $k = 3$; then processing each of the disjuncts proceeds as follows:

   - kkwkww is processed, from left to right, generating the physical plan

     $$I_{G,k}(\mathsf{w^- k^- k^-}) \bowtie I_{G,k}(\mathsf{kww})$$

     in which $\bowtie$ is implemented as a merge join. Note the subexpression kkw has been inverted to obtain the correct sort order to perform a merge join.

   - kkwkwkww is processed from left to right, generating the physical plan

     $$[I_{G,k}(\mathsf{w^- k^- k^-}) \bowtie_1 I_{G,k}(\mathsf{kwk})] \bowtie_2 I_{G,k}(\mathsf{ww})$$

     in which $\bowtie_1$ is implemented as a merge join and $\bowtie_2$ as a hash join.

   - kkwkwkwkww is processed from left to right, generating the physical plan

     $$[(I_{G,k}(\mathsf{w^- k^- k^-}) \bowtie_1 I_{G,k}(\mathsf{kwk}))$$
     $$\bowtie_2 I_{G,k}(\mathsf{wkw})] \bowtie_3 I_{G,k}(\mathsf{w})$$

     in which $\bowtie_1$ is implemented as a merge join and $\bowtie_2$ and $\bowtie_3$ as hash joins.

We term this evaluation strategy *semi-naive*. The complete physical plan is formed as a union of these three sub-plans.

The third step can be optimized by using the histogram $sel_{G,k}$, as follows. For each disjunct D of R''

1. if $|D| \leq k$, return $I_{G,k}(D)$.

2. Find the most selective k-path subquery D' of D (i.e., the k-path with smallest $sel_{G,k}$ value). There are $|D| - (k - 1)$ such subqueries to consider.

3. Let $D = D_{\mathrm{left}} \circ D' \circ D_{\mathrm{right}}$, and recur on D_{left} and D_{right}, to generate query plans for respective output streams $LEFT$ and $RIGHT$.

4. Determine the cost of each of the following alternative query plans, and return the cheapest plan:

   - $[LEFT \bowtie I_{G,k}(D')] \bowtie RIGHT$,
   - $LEFT \bowtie [I_{G,k}(D') \bowtie RIGHT]$,
   - $[LEFT \bowtie I_{G,k}(D'^-)] \bowtie RIGHT$, or
   - $LEFT \bowtie [I_{G,k}(D'^-) \bowtie RIGHT]$.

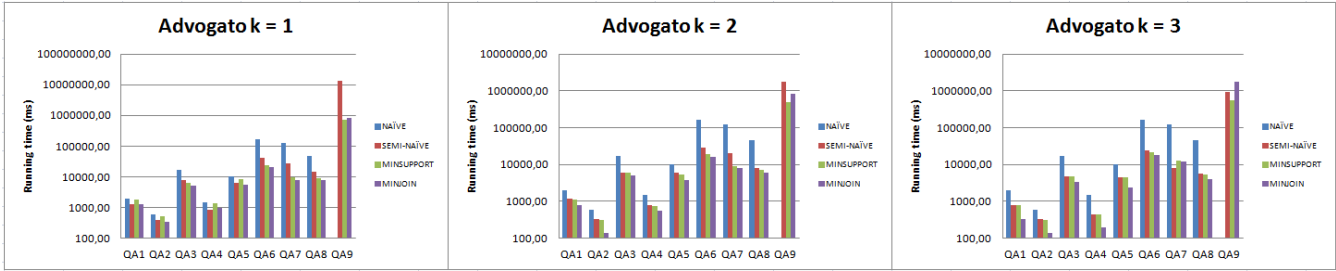We term this evaluation strategy *minSupport*.

Figure 2: Advogato query execution times (ms)

5. EMPIRICAL EVALUATION

We refer the reader to [12] for full details of an empirical evaluation of our system with respect to a broad spectrum of RPQs over four different real and synthetic datasets. In addition to the semi-naive and minSupport evaluation methods described above, two other methods are investigated: *naive*, in which k is fixed at 1 (so indexing is on edge labels, not path labels), which corresponds to automaton-based evaluation (approach 1, discussed in the Introduction); and *minJoin*, which is similar to minSupport but also aims to minimize the number of joins.

As an indicative subset of the empirical results obtained, the three graphs in Figure 2 show the run-times of 8 queries over the Advogato data set, for each of the four evaluation methods, with values of k ranging from 1 to 3. Advogato is a real-world social network having 6,541 nodes and 51,127 edges with $|\mathcal{L}| = 3$, where edges indicate varying degrees of trust between users in the network [11].[4]

We observe that the naive method always performs worst, that the semi-naive method is generally outperformed by minSupport and minJoin, and that the latter two perform similarly. This demonstrates the value of the lightweight histogram data structure for selectivity estimation. We also see that increasing the value of k generally improves the run-times for all methods (apart from naive, where k is fixed at 1 throughout).

6. DEMONSTRATION OVERVIEW

We give participants a hands-on overview of the life of a regular path query, from its submission to our system, through parsing and optimization, to execution. We further demonstrate the speed-ups achieved by our approach compared with Datalog-based evaluation (approach (2), discussed in the Introduction), where our solution is on average 1200x faster on the Advogato queries [12].

Through these interactive activities, we hope to both demonstrate the promise of our approach to RPQ evaluation and to stimulate further broader discussion and study in the research community of engineering strategies for this challenging practical class of graph queries. A system demonstration is an excellent setting in which to accomplish these goals.

7. REFERENCES

[1] E. Bertino et al. Object-oriented databases. In E. Bertino et al, editor, *Indexing Techniques for Advanced Database Systems*, pages 1–38. Kluwer, 1997.

[2] G. Cormode, M. N. Garofalakis, P. J. Haas, and C. Jermaine. Synopses for massive data: Samples, histograms, wavelets, sketches. *Foundations and Trends in Databases*, 4(1-3):1–294, 2012.

[3] S. C. Dey et al. On implementing provenance-aware regular path queries with relational query engines. In *GraphQ*, pages 214–223, Genoa, 2013.

[4] J. Fan, G. S. Raj, and J. M. Patel. The case against specialized graph analytics engines. In *CIDR*, Asilomar, California, 2015.

[5] W. Fan, J. Li, S. Ma, N. Tang, and Y. Wu. Adding regular expressions to graph reachability and pattern queries. *Frontiers of Comp. Sci.*, 6(3):313–338, 2012.

[6] A. Gubichev, S. J. Bedathur, and S. Seufert. Sparqling kleene: fast property paths in RDF-3X. In *GRADES*, New York, NY, 2013.

[7] A. Gubichev and M. Then. Graph pattern matching – do we have to reinvent the wheel? In *GRADES*, Snowbird, Utah, 2014.

[8] S. Harris and A. Seaborne, editors. *SPARQL 1.1 Query Language*, W3C Recomm., 2013.

[9] A. Jindal and S. Madden. GRAPHiQL: A graph intuitive query language for relational databases. In *Big Data*, pages 441–450, Washington, DC, 2014.

[10] A. Koschmieder and U. Leser. Regular path queries on large graphs. In *SSDBM*, pages 177–194, Chania, Crete, Greece, 2012.

[11] P. Massa, M. Salvetti, and D. Tomasoni. Bowling alone and trust decline in social network sites. In *DASC*, pages 658–663, Chengdu, China, 2009.

[12] J. Peters. Regular path query evaluation using path indexes. Master's thesis, Eindhoven University of Technology, 2015.

[13] P. Selmer, A. Poulovassilis, and P. T. Wood. Implementing flexible operators for regular path queries. In *GraphQ*, Brussels, 2015.

[14] J. Sumrall, G. H. L. Fletcher, and A. Poulovassilis et al. Investigations on path indexing for neo4j, 2015. Under review.

[15] K.-F. Wong, J. X. Yu, and N. Tang. Answering XML queries using path-based indexes: A survey. *World Wide Web*, 9(3):277–299, 2006.

[16] P. T. Wood. Query languages for graph databases. *SIGMOD Record*, 41(1):50–60, 2012.

[17] N. Yakovets, P. Godfrey, and J. Gryz. WAVEGUIDE: evaluating SPARQL property path queries. In *EDBT*, pages 525–528, Brussels, 2015.

---

[4]The data set is publicly available at `http://konect.uni-koblenz.de/networks/advogato`